



Oxford Cambridge and RSA

Friday 17 June 2022 – Afternoon

AS Level Further Mathematics B (MEI)

Y414/01 Numerical Methods

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 You are given the following simultaneous equations.

$$2x + 1.8y = 5$$

$$3x + 2.8y = 3$$

The constants and the coefficients of x are exact, but the coefficients of y have been **chopped** to 1 decimal place.

- (a) Calculate the maximum possible relative error in each of the coefficients of y . [2]
- (b) Determine the range of possible values of y . [4]
- (c) Explain why this range is so large. [1]

- 2 The table shows some values of x and the associated values of $y = f(x)$.

x	1.98	1.99	2	2.01	2.02
$f(x)$	1.10311648	1.10514069	1.10714872	1.10914075	1.11111695

- (a) Use the central difference method to calculate **two** approximations to the value of $\frac{dy}{dx}$ at $x = 2$. [3]
- (b) State the value of $\frac{dy}{dx}$ at $x = 2$ as accurately as you can, justifying the precision quoted. [1]
- (c) Calculate an approximation of the **error** in using $f(2)$ to approximate $f(2.008)$. [2]

- 3 Ali uses the trapezium rule with $h = 1$, $h = 0.5$ and $h = 0.25$ to calculate three approximations to $\int_1^2 f(x) dx$. Ali's results are shown in the table.

h	n	T_n
1	1	0.9462734
0.5	2	0.9645336
0.25	4	0.9691932

- (a) Use the information in the table to calculate **two** approximations to $\int_1^2 f(x) dx$ using Simpson's rule, giving your answers correct to **6** decimal places. [3]
- (b) **Without** doing any further calculation, state the value of $\int_1^2 f(x) dx$ as accurately as you can, justifying the precision quoted. [1]

Ali states that the graph of $y = f(x)$ is concave downwards between $x = 1$ and $x = 2$.

- (c) Explain whether the information in the table supports Ali's statement. [1]

- 4 The equation $3x^5 - 13x^2 + 11 = 0$ has three roots, α , β and γ such that $\alpha < \beta < \gamma$.
Fig. 4.1 shows part of the graph of $y = 3x^5 - 13x^2 + 11$.

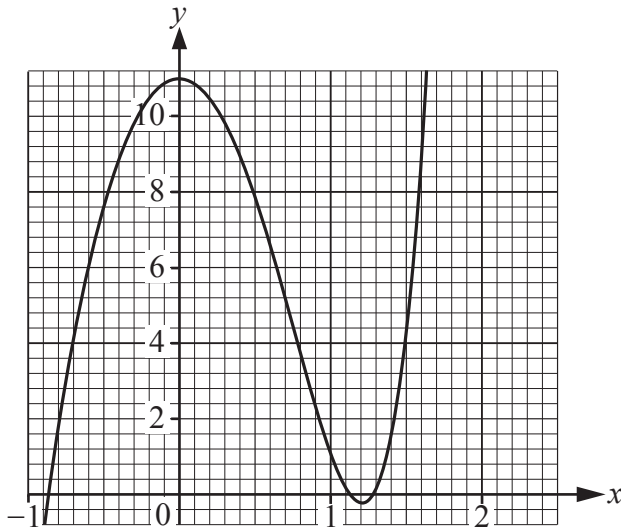


Fig. 4.1

- (a) Explain why it is not possible to use the method of false position with initial values of $a = 1$ and $b = 1.5$ to find β . [1]

Taylor uses the method of false position to find β using initial values of $a = 1$ and $b = 1.2$. The associated spreadsheet output is shown in **Fig. 4.2**.

	C	D	E	F	G	H
3	a	$f(a)$	b	$f(b)$	x_{new}	$f(x_{new})$
4	1	1	1.2	-0.255040	1.159357	-0.189833
5	1	1	1.159357	-0.189833	1.133933	-0.091284
6	1	1	1.133933	-0.091284	1.122729	-0.035016
7	1	1	1.122729	-0.035016	1.118577	-0.012267
8	1	1	1.118577	-0.012267	1.11714	-0.004160
9	1	1	1.117140	-0.004160	1.116655	-0.001395
10	1	1	1.116655	-0.001395	1.116492	-0.000466
11	1	1	1.116492	-0.000466	1.116438	-0.000156
12	1	1	1.116438	-0.000156	1.116420	-5.19E-05

Fig. 4.2

(b) Write down a suitable spreadsheet formula for the following.

(i) cell F4 [1]

(ii) cell G4 [2]

The spreadsheet formula in cell C5 is

`=IF(H4>0,G4,C4)`.

(c) Write down a suitable formula for cell E5. [1]

(d) **Without** doing any further calculation, state the value of β as accurately as you can, justifying your answer. [1]

(e) Show that the Newton-Raphson iteration is

$$x_{n+1} = \frac{12x_n^5 - 13x_n^2 - 11}{15x_n^4 - 26x_n}. \quad [2]$$

(f) Use the Newton-Raphson iteration with $x_0 = -1$ to find α correct to 7 decimal places. [2]

Taylor uses the Newton-Raphson iteration to find γ correct to 7 decimal places. The associated spreadsheet output, together with some further analysis, is shown in **Fig. 4.3**.

r	x_r	difference	ratio
0	1.5		
1	1.3773266	-0.1226734	
2	1.3108192	-0.0665073	0.5421493
3	1.2840785	-0.0267407	0.4020718
4	1.2789338	-0.0051447	0.1923936
5	1.2787404	-0.0001934	0.0375917
6	1.2787401	-2.713E-07	0.0014026
7	1.2787401	-5.329E-13	1.965E-06

Fig. 4.3

(g) Explain what the values in the ratio column in **Fig. 4.3** tell you about the convergence of this sequence of estimates to γ . [2]

- 5 The equation $x^5 - 3x + 1 = 0$ has two positive roots, α and β , where $\alpha < \beta$.

Charlie is using the iterative formula $x_{n+1} = g(x_n)$ where $g(x_n) = \frac{x_n^5 + 1}{3}$ to try to find α and β .

Fig. 5.1 shows part of the graphs of $y = x$ and $y = g(x)$.

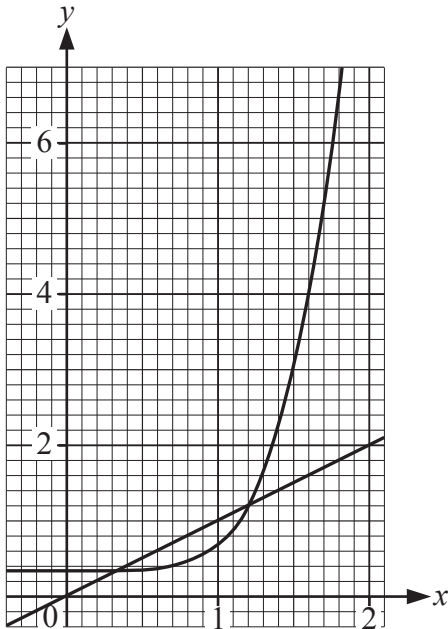


Fig. 5.1

- (a) With reference to **Fig. 5.1**, explain why Charlie's iterative formula may be successfully used to find α , but will not find β . [2]
- (b) Use the iterative formula with a starting value of $x_0 = 1$ to find α correct to 6 decimal places. [2]

Charlie uses the iterative formula to try to find β . Her spreadsheet output is shown in **Fig. 5.2**.

r	x_r
0	1.5
1	2.8645833
2	64.629643
3	375869894
4	2.501E+42
5	3.26E+211
6	#NUM!

Fig. 5.2

- (c) Explain why the spreadsheet displays #NUM! for the value of x_6 . [1]
- (d) Use the relaxed iteration $x_{n+1} = (1 - \lambda)x_n + \lambda g(x_n)$, with $\lambda = -0.33$ and $x_0 = 1.5$ to find β correct to 6 decimal places. [2]

6 **Fig. 6.1** shows some values of x and the associated values of $f(x)$.

x	1	1.25	1.5	1.75	2
$f(x)$	0	0.1609640	0.2924813	0.4036775	0.5

Fig. 6.1

Sam is using the midpoint rule to find a sequence of estimates to $\int_1^2 f(x) dx$. Some of Sam's spreadsheet output is shown in **Fig. 6.2**.

n	M_n	difference	ratio
1			
2			
4	0.2795859	-0.0027349	0.2691687
8	0.2788869	-0.0006989	0.2555667
16	0.2787112	-0.0001758	0.2514659
32	0.2786672	-4.401E-05	0.2503719
64	0.2786561	-1.101E-05	0.2500933

Fig. 6.2

- (a) Use the information in **Fig. 6.1** to fill in the **three** missing values on the copy of **Fig. 6.2** in the Printed Answer Booklet, giving your answers correct to 7 decimal places. [4]
- (b) Explain why the last difference is displayed as $-1.101E-05$ **not** $-1.1E-05$. [1]
- (c) Use extrapolation to determine the value of $\int_1^2 f(x) dx$ as accurately as you can, justifying the precision quoted. [5]

- 7 Azmi placed a container of liquid in a refrigerator unit in a laboratory. Initially the temperature of the liquid was 22°C .

Azmi recorded the temperature, $y^{\circ}\text{C}$, of the liquid t minutes after putting the liquid in the refrigerator unit.

The results are shown in the table.

t	0	6	10
y	22	8.824	3

Azmi believes that the data may be modelled by a polynomial.

- (a) Explain why it is not possible to use Newton's forward difference interpolation method for these data. [1]
- (b) Use Lagrange's form of the interpolating polynomial to construct a quadratic model for these data. [4]

Later Azmi finds that when $t = 20$, $y = -4$.

- (c) Determine whether the quadratic model is a good fit for these values. [2]

Azmi uses all the data available to construct the cubic model

$$y = at^3 + bt^2 - 2.7t + 22, \text{ where } a \text{ and } b \text{ are constants.}$$

- (d) Use Lagrange's method to determine the values of a and b . [4]

The background temperature in the refrigerator unit is -5°C .

- (e) Verify that, according to the cubic model, the liquid has reached the background temperature after 30 minutes. [1]
- (f) Adapt this model so that it would be suitable for large values of t . [1]

END OF QUESTION PAPER

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